

# Progressive image coding based on an adaptive block compressed sensing

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**Abstract:** This paper proposes an adaptive block-based compressed sensing (ABCS) technique to build a new progressive image coding scheme, in which both image acquisition and reconstruction are carried out in two layers. At the base layer, an original image is sampled and restored by the block-based compressed sensing (BCS) method with a low and fixed measurement rate. Second, all blocks in the enhancement layer are re-sampled with different rates according to a block classification. The final reconstruction of a block at the enhancement layer is performed in multiple stages where each stage only knows a part of sampled coefficients. We present some experimental results to show that our proposed ABCS method outperforms the BCS method; in particular, it produces a better visual quality in regions that contain edges, patterns, and textures.

**Keywords:** compressed sensing, adaptive sampling, progressive coding

**Classification:** Science and engineering for electronics

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## 1 Introduction

The past few years have witnessed a widespread interest of a new signal processing theory called the compressed sensing (CS) [1], which can sample and compress signals simultaneously at a sub-*Nyquist* rate while offering a highly precise reconstruction. One big advantage of the CS technique is that it makes acquisition of digital data much simpler.

When applied to 2-D images, however, CS faces several challenges, e.g., a huge memory is needed to store the measurement matrix and the reconstruction process is very complex. To address these challenges, several fast algorithms (e.g., [2]) have been developed for the CS reconstruction, while a block-based compressed sensing (BCS) is proposed in [3] to reduce the memory requirement. Since the BCS method only needs to store a small measurement matrix, it is especially suitable for real-time 2-D image applications. The performance can be further improved by applying some directional transforms to BCS so as to increase the sparseness [4]. The block-based image acquisition in these BCS methods is implemented with the same measurement operator, i.e., each block is sampled with a fixed measurement rate, ignoring the diversified contents in various blocks that would eventually influence the reconstruction quality.

On the other hand, many applications require a progressive coding of image data. When channel congestion occurs for instance, the sampled data should be divided into several segments, each being transmitted in disparate time slots (rather than transmitting the entire data in one slot). In this way, the restored image becomes clearer when more segments are received.

Based on the above discussions, we extend the BCS method to an adaptive block-based compressed sensing (ABCS) and apply the ABCS method in a two-layer structure to achieve the progressive coding. To this end, we will first sample each original image and do the reconstruction by the BCS method (with a low and fixed measurement-rate) to generate the base-layer image. Then, the enhancement layer is obtained between this reconstructed version and the original image. We propose to perform a block classification at the enhancement layer and then apply the ABCS scheme (i.e., to sample different image blocks at different rates, instead of using the same rate as in BCS). Obviously, the base layer provides an image at a pretty low quality. We propose to do the reconstruction at the enhancement layer in multiple stages (where each stages only knows a part of sampled coefficients) so as to improve the overall quality progressively.

## 2 Background

In the CS theory, let's use  $\mathbf{x}$  to denote a real-valued signal vector with length  $N$ . Let's assume that an  $N \times N$  basis matrix  $\Psi$  provides a  $K$ -sparse representation for  $\mathbf{x}$ , i.e.,  $\mathbf{x} = \Psi \cdot \boldsymbol{\theta}$ , where the coefficient vector  $\boldsymbol{\theta}$  has  $K$  significant values. Then,  $\mathbf{x}$  can be reconstructed (with a certain accuracy) by  $M$  measurements:

$$\mathbf{y} = \Phi \cdot \mathbf{x} = \Phi \cdot (\Psi \cdot \boldsymbol{\theta}), \quad (1)$$

where  $\mathbf{y}$  denotes the measurement-vector with length  $M$ , and  $\Phi$  is an  $M \times N$  measurement matrix that is incoherent with  $\Psi$ ,  $M = O(K \log N / K)$ ,  $K < M \ll N$ . In the CS framework,  $\hat{\boldsymbol{\theta}}$  is first recovered from  $\mathbf{y}$  by solving a convex problem (e.g., BP [1]) or an iterative greedy pursuing (e.g., OMP [5] and its successors). Then,  $\mathbf{x}$  can be reconstructed via  $\hat{\mathbf{x}} = \Psi \cdot \hat{\boldsymbol{\theta}}$ .

In the BCS method, suppose that an original image has  $N$  pixels with  $M$  measurements taken. The image is partitioned into  $B \times B$  blocks and each block is sampled with the same operator. Let's assume that each image block  $\mathbf{x}_i$  is taken by  $M_B$  measurements:

$$\mathbf{y}_i = \Phi_B \cdot \mathbf{x}_i, \quad (2)$$

where  $\Phi_B$  is an  $M_B \times B^2$  measurement matrix with  $M_B = \lfloor \frac{M}{N} \cdot B^2 \rfloor$ . The equivalent measurement matrix  $\Phi$  for the entire image is thus a block-wise diagonal one:

$$\Phi = \begin{bmatrix} \Phi_B & & & \\ & \Phi_B & & \\ & & \ddots & \\ & & & \Phi_B \end{bmatrix}. \quad (3)$$

It is clear that only a small measurement matrix with size  $M_B \times B^2$  is needed to store in the BCS method, suggesting a huge memory saving.

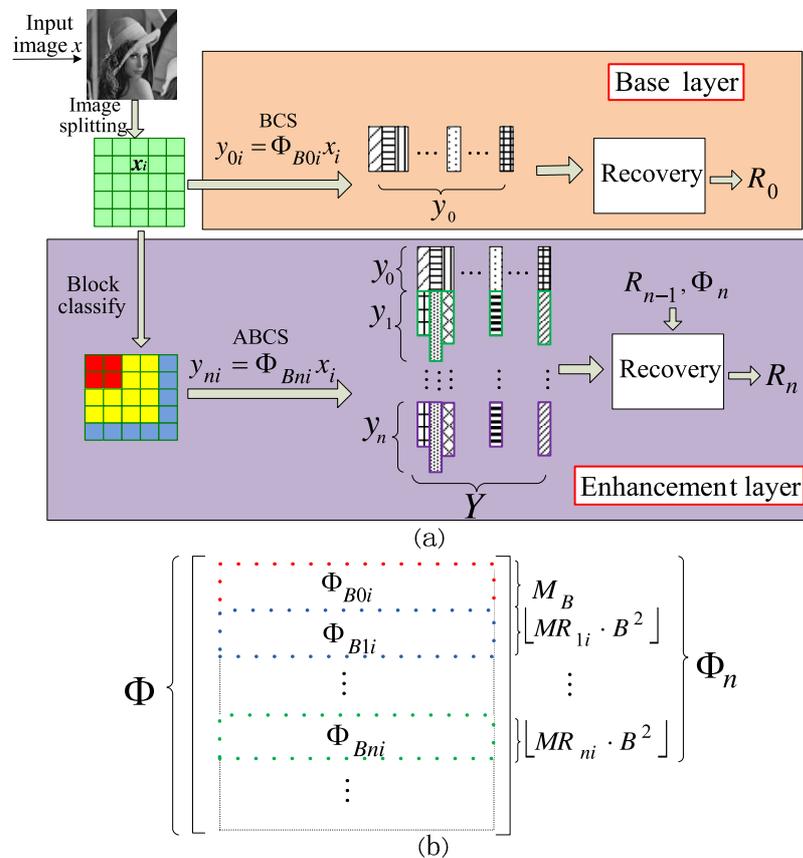


Fig. 1. The progressive image coding based on ABCS

### 3 Progressive image coding based on ABCS

Our proposed progressive image coding scheme based on the ABCS technique is shown in Fig. 1 (a). First, a low-quality image  $R_0$  is recovered by the BCS method with a low and fixed measurement rate – regarded as the base layer. At the enhancement layer, this low-quality image is improved progressively when more stages of processing on each block in the enhancement layer are completed.

#### 3.1 Image block classification

We propose to do a block classification in the enhancement layer: each image block is analyzed and then classified as SMOOTH, TEXTURE, or OTHER. Each of them has different characteristics and thus will be assigned with different measurement rate accordingly. Here, we compute the normalized variance of each block to do the block classification:

$$d = \frac{1}{B \times B} \sum_{j=1}^{B \times B} (p_j - \bar{p})^2, \quad \text{where } \bar{p} = \frac{1}{B \times B} \sum_{j=1}^{B \times B} p_j. \quad (4)$$

$$d' = \frac{d - d_{min}}{d_{max} - d_{min}}, \quad (5)$$

where  $p_j$  denotes the  $j$ -th pixel of the block,  $d_{max}$  and  $d_{min}$  is the maximum and minimum of  $d$  among all the blocks. Accordingly, each block is classified as:

$$\mathbf{x}_i \in \begin{cases} \text{SMOOTH,} & \text{if } d' \leq T_1; \\ \text{TEXTURE,} & \text{if } d' > T_2; \\ \text{OTHER,} & \text{if } T_1 < d' \leq T_2. \end{cases} \quad (6)$$

$T_1$  and  $T_2$  are determined by experiments. In our experiments, we set them at  $T_1 = 0.1$  and  $T_2 = 0.3$ , respectively.

#### 3.2 Adaptive block compressed sensing

According to the block classification, we propose to allocate different rates to measure blocks belonging to different classes so as to construct an adaptive method. For instance, TEXTURE-type blocks are assigned with a high rate to improve these special regions; SMOOTH-type a low rate; and OTHER-type an intermediate rate. The resulted ABCS method can be described as follows:

$$\mathbf{y}_{ni} = \Phi_{Bni} \cdot \mathbf{x}_i, \quad (7)$$

where  $\Phi_{Bni}$  is the measurement matrix imposed to the  $i$ -th block  $\mathbf{x}_i$  in the  $n$ -th stage, with a measurement rate according to (6).

The measurement matrix is created by an orthonormalized i.i.d Gaussian matrix, as was done in [3]. Assuming that  $\Phi$  is a  $B^2 \times B^2$  orthonormalized i.i.d Gaussian matrix, we generate each block measurement matrix  $\Phi_{Bni}$  from  $\Phi$ . An example for generating the measurement matrix for  $\mathbf{x}_i$  is shown in Fig. 1 (b), where  $\Phi_{B0i}$  used in the base layer is generated by the first  $M_B$  rows of  $\Phi$ . And  $\Phi_{B1i}$  used in the first stage of the enhancement layer is generated by the immediate rows from  $M_B + 1$  to  $M_B + \lfloor MR_{1i} \cdot B^2 \rfloor$  of  $\Phi$ , where  $MR_{1i}$

is the corresponding measurement rate. Similarly,  $\Phi_{B2i}$  used in the second stage is generated by the immediate rows from  $M_B + \lfloor MR_{1i} \cdot B^2 \rfloor + 1$  to  $M_B + \lfloor MR_{1i} \cdot B^2 \rfloor + \lfloor MR_{2i} \cdot B^2 \rfloor$  of  $\Phi$ , etc.

### 3.3 Progressive reconstruction

As shown in Fig. 1 (a), the base-layer image can always be recovered by the BCS reconstruction algorithm at the decoder side to offer the first approximation. Then, in the enhancement layer, for the block  $\mathbf{x}_i$  in its  $n$ -th stage of reconstruction, we combine  $\mathbf{y}_{ni}$  with the previous measurements  $\mathbf{y}_{mi}$  ( $m = 0, 1, 2, \dots, n - 1$ ) to generate a measurement vector  $\mathbf{Y}_i$  (the  $i$ -th column vector of  $\mathbf{Y}$  in Fig. 1 (a)). Meanwhile,  $\Phi_{Bni}$  is combined with previous measurement matrices  $\Phi_{Bmi}$  ( $m = 0, 1, 2, \dots, n - 1$ ) to create  $\Phi_n$  as shown in Fig. 1 (b). Finally, both  $\mathbf{Y}_i$  and  $\Phi_n$  are inputted to the BCS reconstruction algorithm to improve the image recovered in the previous stage. Here, since we can record the block measurement rates in each stage, the measurement matrices can be obtained from  $\Phi_0$  based on the recorded measurement rates, instead of storing each  $\Phi_{Bni}$ . Thus, the extra memory expense is negligible. Additionally, the recovered image in the previous stage is used as the initial solution for the current stage, which has speeded up the reconstruction of the enhancement layer.

With the stage-by-stage reconstruction at the enhancement layer, we get a progressively improved image. Because a higher rate is assigned to TEXTURE-type blocks, we can end up with a better quality in these special regions.

## 4 Experimental results

To validate the performance of our ABCS method, we compare it with the BCS method proposed in [4] for the case of only applying contourlet transforms. For a fair comparison, we calculate the equivalent measurement rate (EMR), whenever the enhancement layer is transmitted. The measurement rate used in BCS is the same as EMR. The fixed measurement rate used at the base layer is 0.1; and the block size  $B = 32$  is employed as suggested in [3].

Table I tabulates the PSNR comparison of four  $256 \times 256$  images *Lena*, *Boat*, *Cameraman* and *Barbara*. It is seen that our method achieves a superiority of 0.12 dB, 0.21 dB, 1.50 dB, and 0.56 dB in PSNR on average over BCS, respectively. The improvement for *Cameraman* is the highest. This is mainly because there are more smooth regions in *Cameraman* which can be easily recovered even with a low measurement rate. Meanwhile, the averaged PSNR of all TEXTURE blocks of four images are also given, from which we can find that our method yields a very significant improvement for *Lena*, *Boat*, and *Cameraman* (2.38 dB, 2.04 dB, and 3.47 dB on average, respectively). For *Barbara* (which contains more TEXTURE blocks), however, the improvement is only 0.58 dB on average.

Fig. 2 shows the reconstructed images for *Lena* with  $\text{EMR} = 0.289$ . As

**Table I.** PSNR comparison of ABCS and BCS (in dB)

EMR			0.1	0.206	0.373	0.539	0.706
Lena	PSNR	ABCS	24.25	27.26	30.20	32.88	35.77
		BCS		27.05	30.14	32.75	35.68
	Mean PSNR of TEXTURE blocks	ABCS	24.03	30.06	33.29	36.34	40.38
		BCS		27.53	31.38	34.25	37.38
EMR			0.1	0.219	0.386	0.553	0.720
Boat	PSNR	ABCS	24.30	27.55	30.79	33.87	37.32
		BCS		27.38	30.62	33.62	37.07
	Mean PSNR of TEXTURE blocks	ABCS	24.04	29.11	32.20	35.41	39.63
		BCS		27.26	30.52	33.54	36.86
EMR			0.1	0.172	0.332	0.492	0.652
Cameraman	PSNR	ABCS	22.29	25.64	28.72	31.42	34.23
		BCS		23.96	27.29	30.00	32.75
	Mean PSNR of TEXTURE blocks	ABCS	19.52	24.85	28.23	31.61	35.88
		BCS		21.49	25.29	28.40	31.51
EMR			0.1	0.270	0.442	0.615	0.788
Barbara	PSNR	ABCS	20.02	22.10	23.83	25.86	28.93
		BCS		21.93	23.59	25.50	28.37
	Mean PSNR of TEXTURE blocks	ABCS	19.84	22.17	23.95	26.04	29.30
		BCS		21.81	23.54	25.47	28.32



**Fig. 2.** Recovered image Lena ( $256 \times 256$ ). (a): original image; (b): using BCS, PSNR = 28.68 dB; (c): using proposed ABCS, PSNR = 28.90 dB; (a0), (a1), (a2): original image blocks got from (a), cheek, eye and shoulder, respectively; (b0), (b1), (b2): got from (b), PSNR = 25.61, 27.65, 28.81 dB, respectively; (c0), (c1), (c2): got from (c), PSNR = 27.31, 29.90, 31.41 dB, respectively.

can be seen, though the PSNR values of Fig. 2 (b) and (c) are similar, our ABCS method achieves a better visual quality at some special regions (e.g., hair), which enables a better visual sense for the recovered image.

Meanwhile, some typical TEXTURE blocks got from Fig. 2 (a), (b), and (c), are also illustrated in Fig. 2. As can be seen, for the blocks of cheek, eye, and shoulder, the proposed ABCS method outperforms BCS by 1.70 dB, 2.25 dB, and 2.60 dB PSNR improvements, respectively.

## 5 Conclusions

In this paper, a progressive image coding scheme based on an adaptive block-based compressed sensing (ABCS) has been proposed. The progressive feature is achieved by a 2-layer (the base layer and enhancement layer) structure. A block classification is performed at the enhancement layer in order to yield an adaptive compressed sensing of individual blocks and the sensing measurements are put into groups so as to facilitate a stage-by-stage processing (the processing at one stage assumes that a new group of sensing measurements is received). At the decoder-side, the base-layer image (with a pretty low quality) is first obtained with the traditional BCS method. Then, all received measurements in the enhancement layer are combined to enhance the base-layer image progressively. Experimental results show that, as compared to the BCS method, our proposed method achieves a similar reconstruction quality over each whole 2-D image but a much better quality in all texture-type regions.

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